INTRODUCTION TO ELECTRIC MACHINES

1 **Magnetic circuits theory**

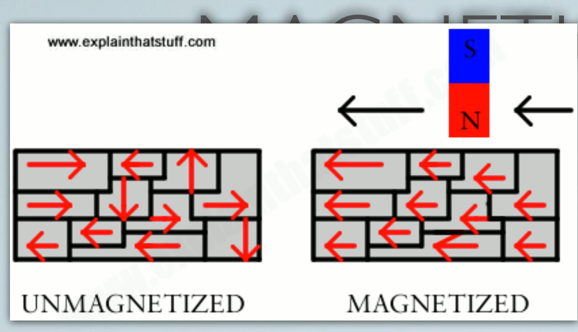
2 Transformers

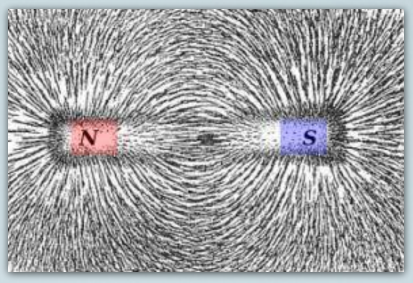
Most pictures come from Jesús Fraile Mora "Máquinas Eléctricas" 5ª ed. 2003, McGraw Hill

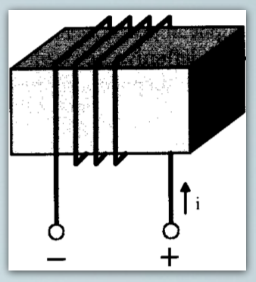
1 MAGNETIC CIRCUITS THEORY

- **Magnetic materials**
- Magnetic circuits
- Magnetic energy
- Alternative current and magnetism

MAGNETIC MATERIALS







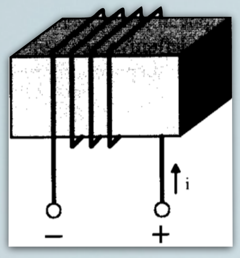
- Magnetized materials
- H : magnetic field intensity (A/m)
- B : magnetic induction (T Tesla)
- μ : Magnetic permeability (H/m)

$$\vec{B} = \mu \, \vec{H}$$

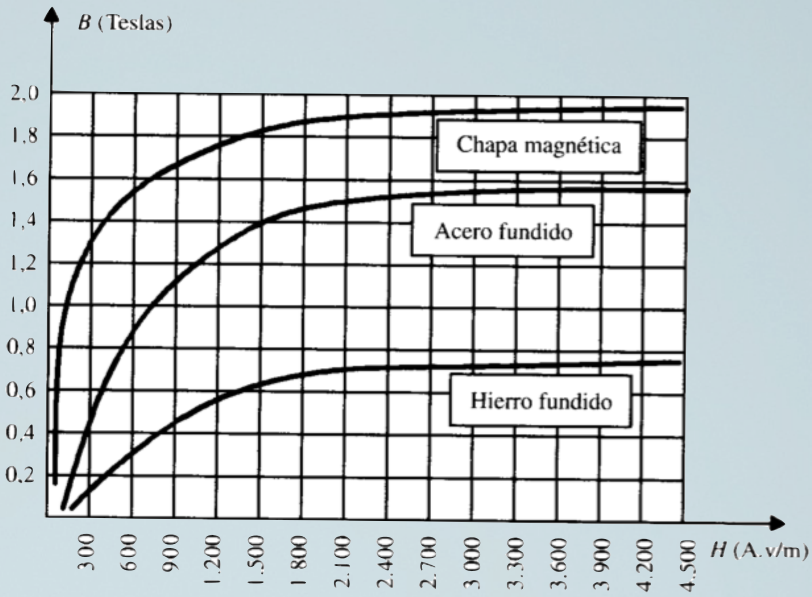
$$\mu = \mu_0 \, \mu_r$$

- Diamagnetic $0 \leq \mu_r < 1$
- Paramagnetic $\mu_r \approx 1000$
- Ferromagnetic $\mu_r \gg 1000$

MAGNETIC MATERIALS

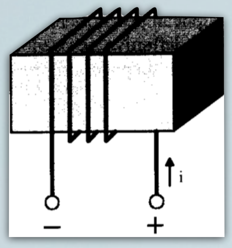


$$\mu = \frac{B}{H}$$

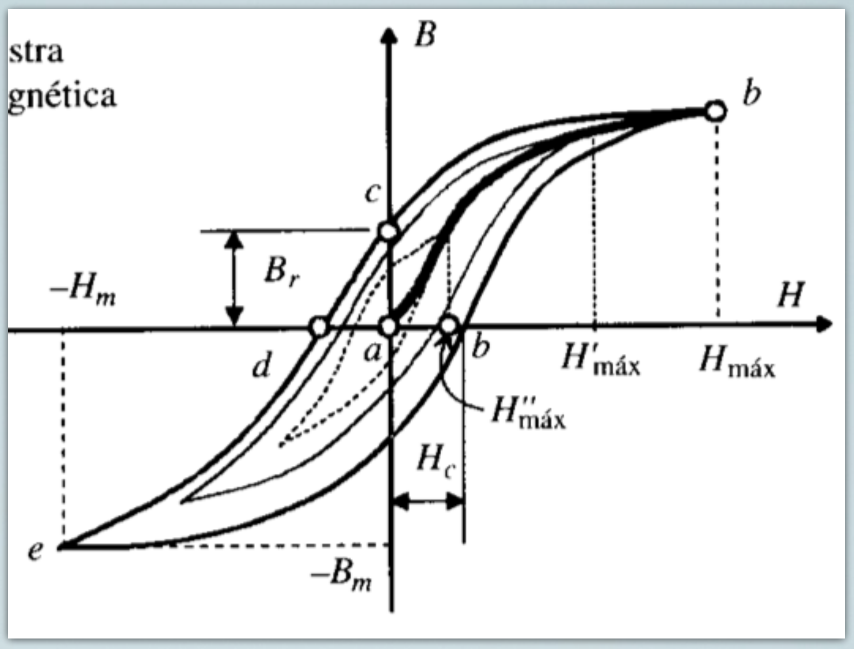


Magnetization curves of different ferromagnetic materials

MAGNETIC MATERIALS



$$\mu = \frac{B}{H}$$



Magnetic hysteresis

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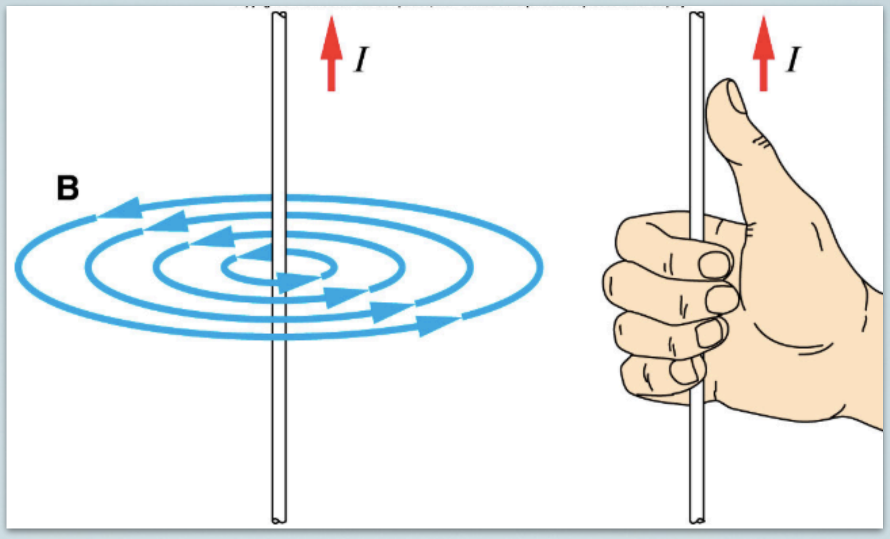
MAGNETIC CIRCUITS THEORY

- Magnetic materials
- **Magnetic circuits**
- Magnetic energy
- Alternative current and magnetism

MAGNETIC CIRCUITS

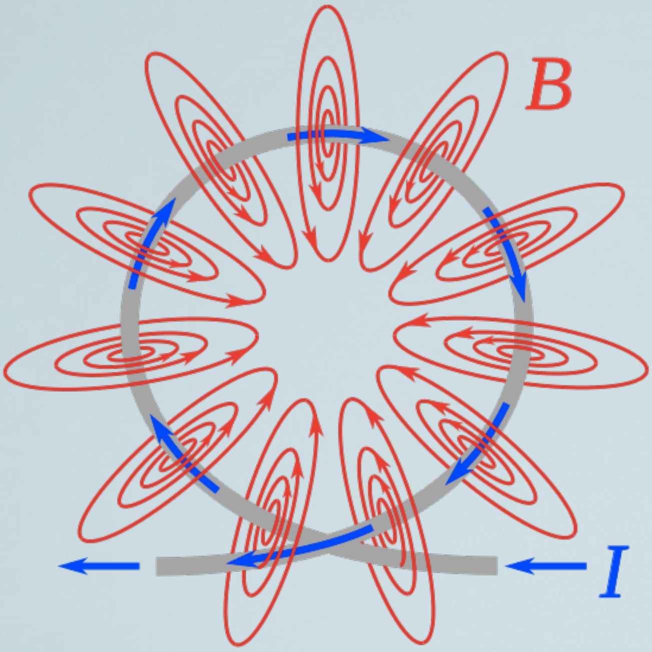
Ampere's law:

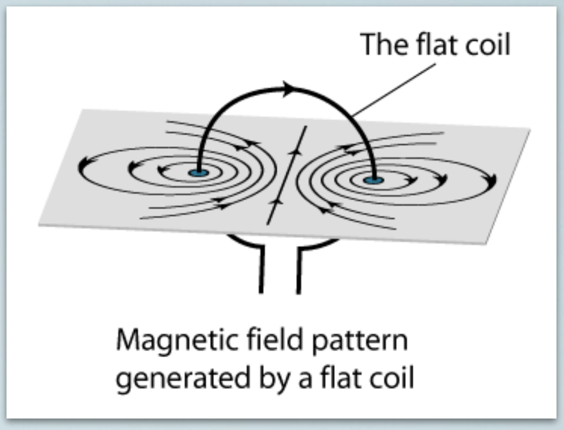


Every electric current ***I*** creates a magnetic field ***B***

MAGNETIC CIRCUITS

Ampere's law:

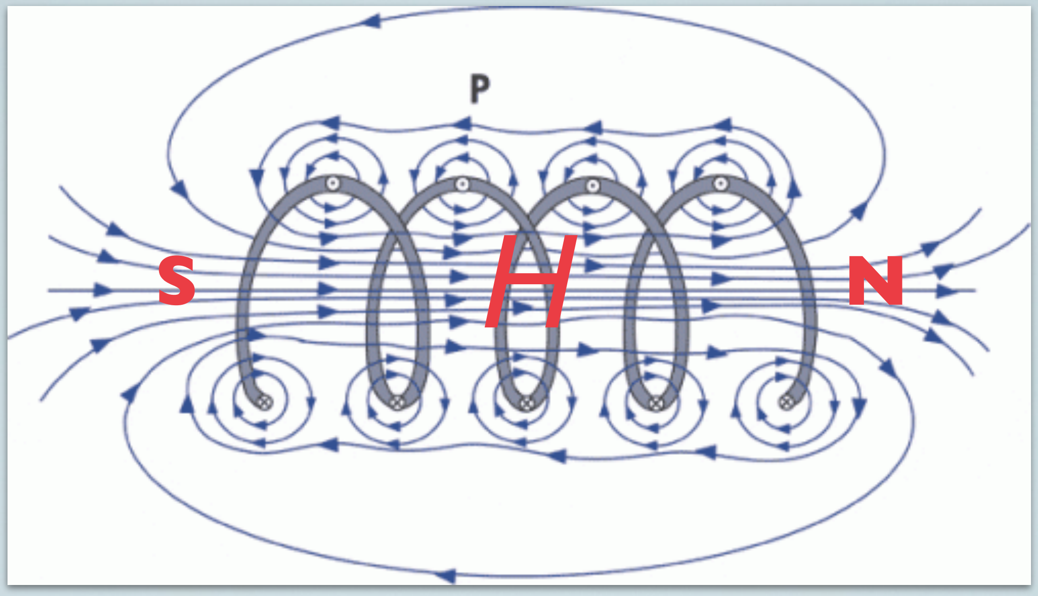




Magnetic field created by a loop wired current

MAGNETIC CIRCUITS

Ampere's law:



Magnetic field created by a solenoid (coil)

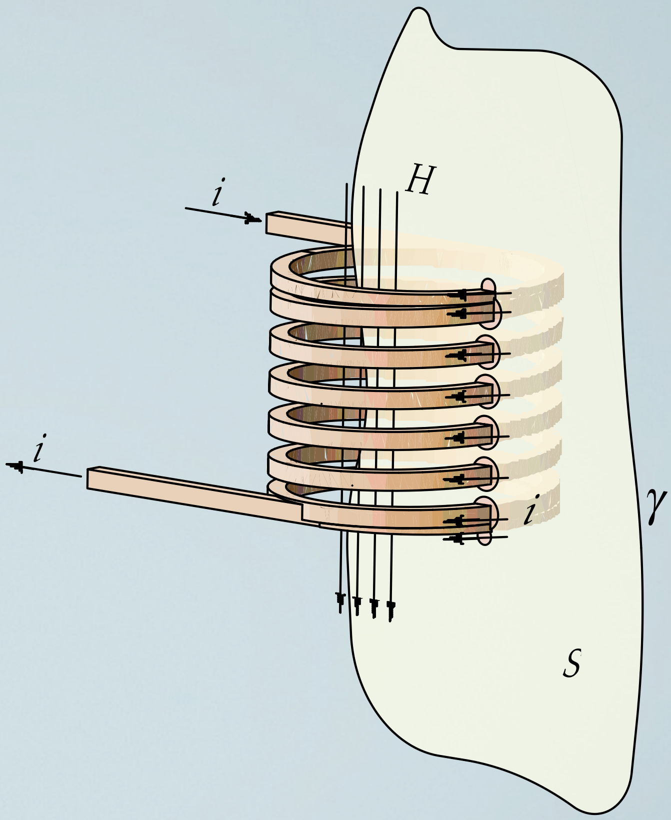
MAGNETIC CIRCUITS

Ampere's law:

$$\oint_{\gamma} \vec{H}\cdot d\vec{\ell} = \int_{S} \vec{J}\cdot d\vec{s}$$

$$H\,\ell = N\,i \quad [A \cdot v]$$

Circulation of magnetic field *H* along a curve



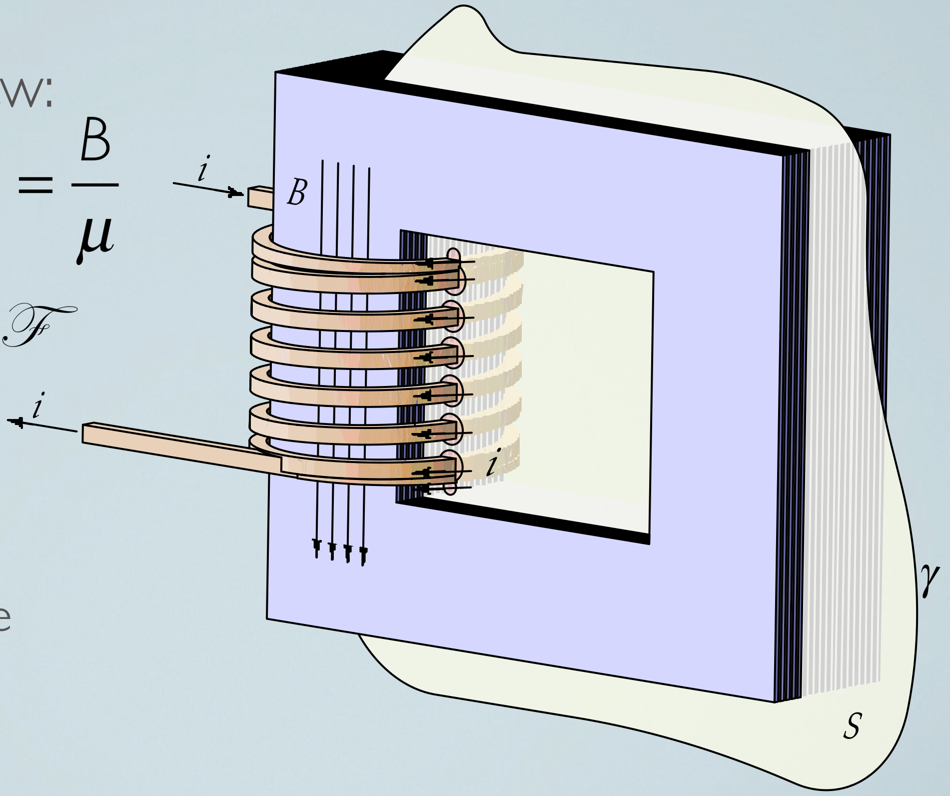
MAGNETIC CIRCUITS

Ampere's law:

$$B = \mu H; \quad H = \frac{B}{\mu}$$

$$\frac{B}{\mu}\ell = N\,i = \mathscr{F}$$

Magnetomotive force (mmf)

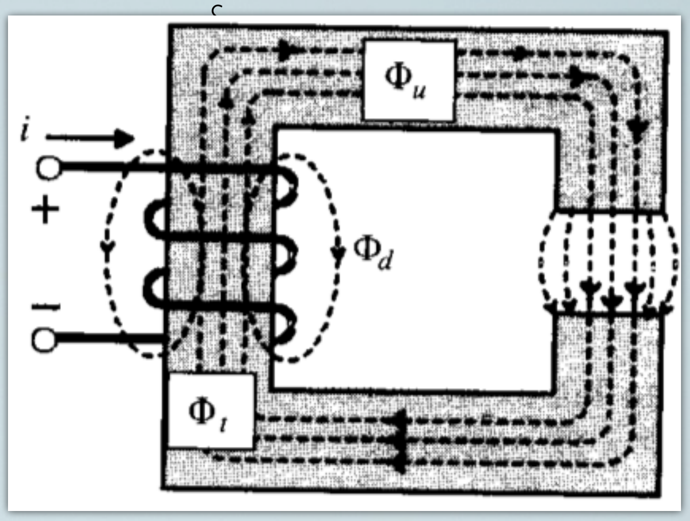


MAGNETIC CIRCUITS

Magnetic flux through S: $\Phi = \int_S \vec{B}\,d\vec{s} \approx B\,S \quad [\mathrm{Wb}]$

Number of magnetic force lines through any perpendicular surface S

Weber



Magnetic flux along a ferromagnetic core

Hopkinson's coefficient:

$$\Phi_t = \Phi_u + \Phi_d$$

$$\upsilon = \frac{\Phi_t}{\Phi_u} = \frac{\Phi_u + \Phi_d}{\Phi_u} = 1 + \frac{\Phi_d}{\Phi_u}$$

MAGNETIC CIRCUITS

Combining equations:

$$\Phi = \int_S \vec{B}\, d\vec{s} \approx B\, S \quad [\mathrm{Wb}]$$

$$\frac{B}{\mu}\, \ell = N\, i = \mathscr{F} \quad [\mathrm{A \cdot v}]$$

$$\mathscr{F} = H\, \ell = \frac{B}{\mu}\, \ell = \Phi \frac{\ell}{\mu\, S} = N\, i \quad [\mathrm{A{\cdot}v}]$$

Magnetic reluctance

$$\mathscr{R} = \frac{\ell}{\mu\, S} \quad [\mathrm{H}^{-1}]$$

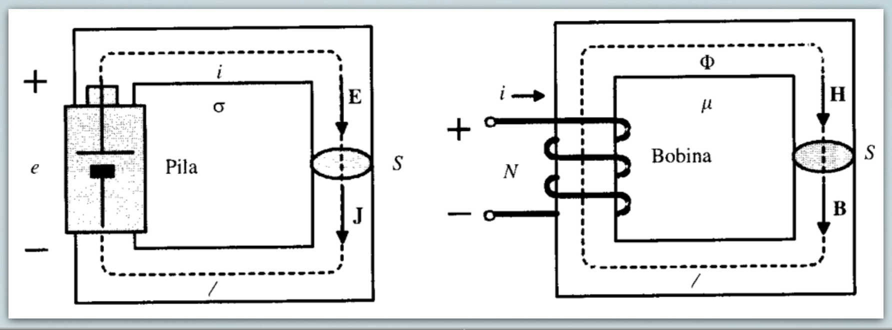
Permeability

$$\mathscr{P} = \frac{1}{\mathscr{R}} = \frac{\mu\, S}{\ell} \quad [\mathrm{H}]$$

Hopkinson's law

$$\mathscr{F} = \Phi \mathscr{R} \quad [\mathrm{A{\cdot}v}]$$

MAGNETIC CIRCUITS



Electric circuit	Magnetic circuit
e : e.m.f.	F : m.m.f.
j : current density (A/m)	B : induction (T)
σ : conductivity (S/m)	μ : permeability (H/m)
i : electric current (A)	Φ : magnetic flux (Wb)
R : electric resistance (Ω)	$\mathscr{R}$: magnetic reluctance (H^{-1})
E : electric field (V/m)	H : magnetic field (A/m)

MAGNETIC CIRCUITS

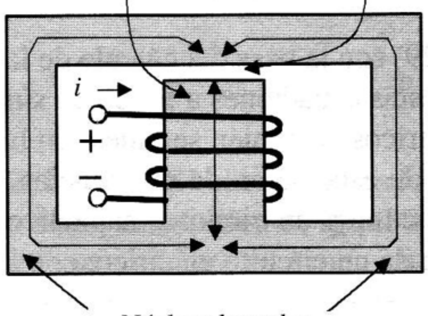
Magnitude	Units
H : intensidad de campo magnético	A/m
Φ : flujo magnético o número de líneas de fuerza que atraviesan cada unidad de superficie perpendicular a ellas	$\Phi = B\ S \cos\alpha$ Weber Wb
μ : permeabilidad magnética	H/m = (Wb/A) / m
B : inducción magnética o densidad de flujo magnético	$B = \mu H$ Tesla T = H/m · A/m = = (Wb/A)/m · A/m = Wb/m^2 1 T = 10 000 Gauss

MAGNETIC CIRCUITS

El núcleo central del circuito magnético de la Figura 1.7 está bobinado con 800 espiras. El material es acero fundido con un valor de la permeabilidad relativa μ_r = 1.000. Calcular la corriente i que debe aplicarse a la bobina para obtener en el entrehierro un flujo de 1 mWb.

Núcleo central
(l = 250 mm; s = 4.000 mm^2)

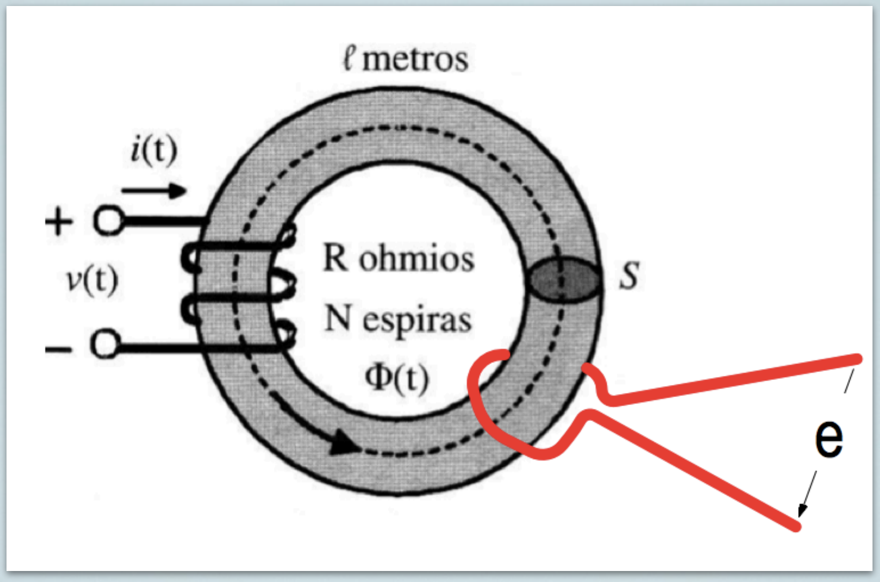
Entrehierro
(g = 1 mm; s = 4.000 mm^2)



Núcleos laterales
(l = 700 mm; s = 2.000 mm^2)

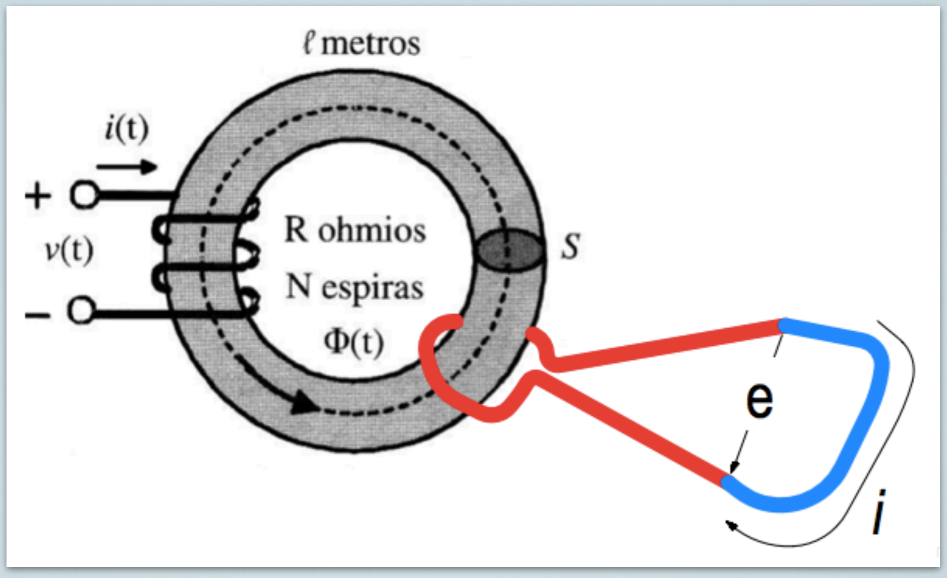
MAGNETIC CIRCUITS

Faraday's law: $e = -N\dfrac{d\Phi}{dt}$ [V]



MAGNETIC CIRCUITS

Faraday's law: $e = -N\dfrac{d\Phi}{dt}$ [V]



MAGNETIC CIRCUITS

Faraday's law: $e = -N\dfrac{d\Phi}{dt} \quad [V]$

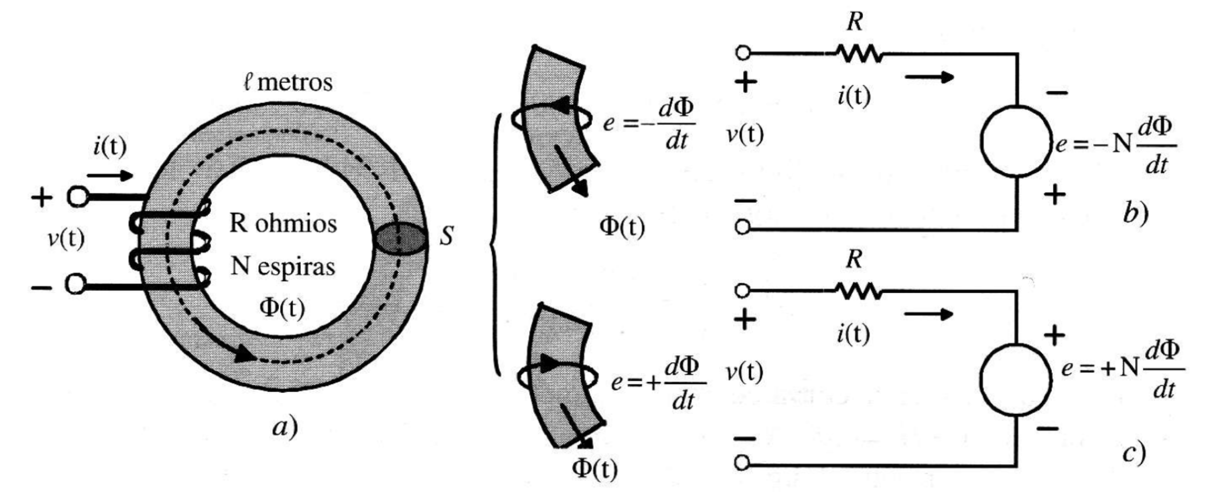


Figura 1.14. F.e.m. y f.c.e.m. inducida en una bobina con núcleo de hierro.

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MAGNETIC CIRCUITS THEORY

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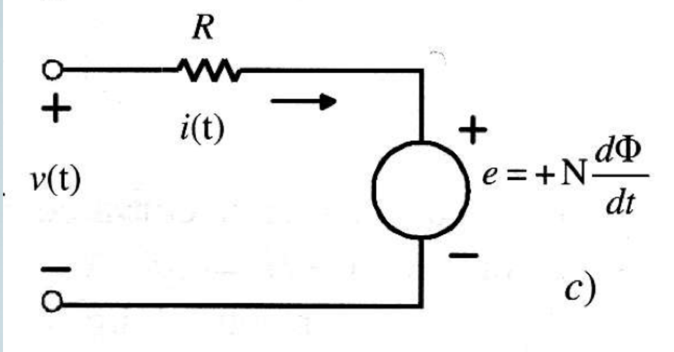
MAGNETIC ENERGY

Energy:



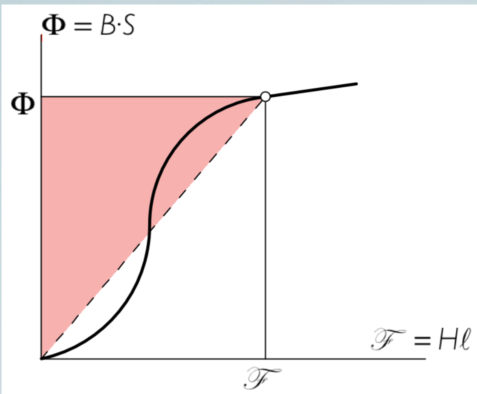
$$v = Ri + N\frac{d\Phi}{dt}; \qquad vi\ dt = Ri^2 dt + Ni\ d\Phi$$

$$dW_e = dW_R + dW_m$$ Steady state (no movement)

$$dW_m = Ni\ d\Phi = \mathcal{F}\ d\Phi; \qquad W_m = \int_{-\Phi}^{\Phi} \mathcal{F}\, d\Phi$$

MAGNETIC ENERGY

Hysteresys:



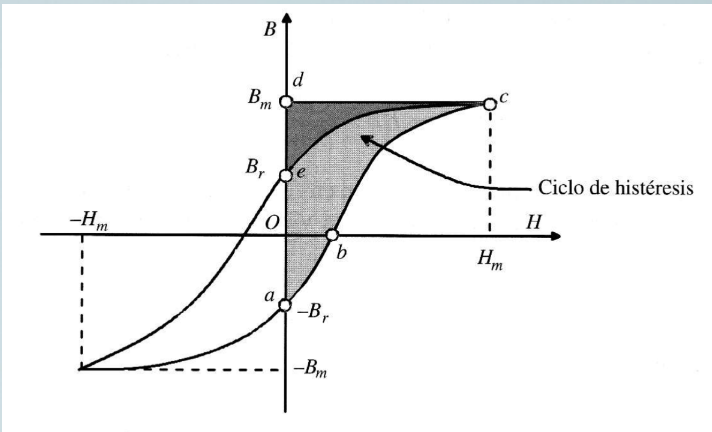


Figura 1.16. Áreas del ciclo de histéresis.

$$W_m = \int_{-\Phi}^{\Phi} \mathcal{F}\, d\Phi = \int_{-B}^{B} H\ \ell\ S\ dB = \ell\ S \int_{-B}^{B} H\ dB$$

$$W_H = vol \int_{-B}^{B} H\ dB; \qquad P_H = f\ W_H; \quad P_H \simeq k_H f\ vol\ B_m^{\alpha}$$

MAGNETIC ENERGY

Other core losses:

eddy currents (Foucault)

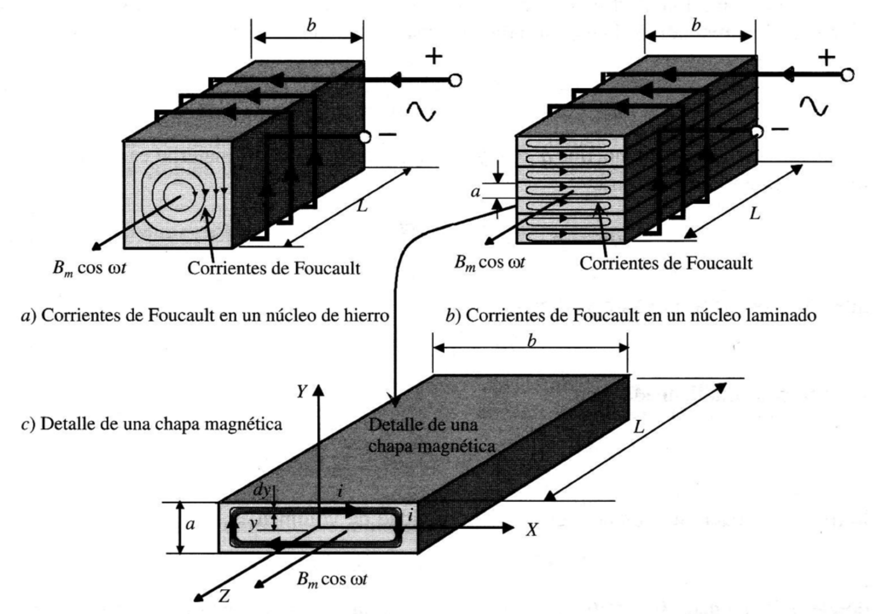


Figura 1.17. Corrientes de Foucault.

$$P_F = f^2 \, k_F \, vol \, B_m^2 a^2 \sigma \qquad P_{Fe} = P_H + P_F = f\,M + f^2\,N$$

MAGNETIC ENERGY

Eddy current looses:

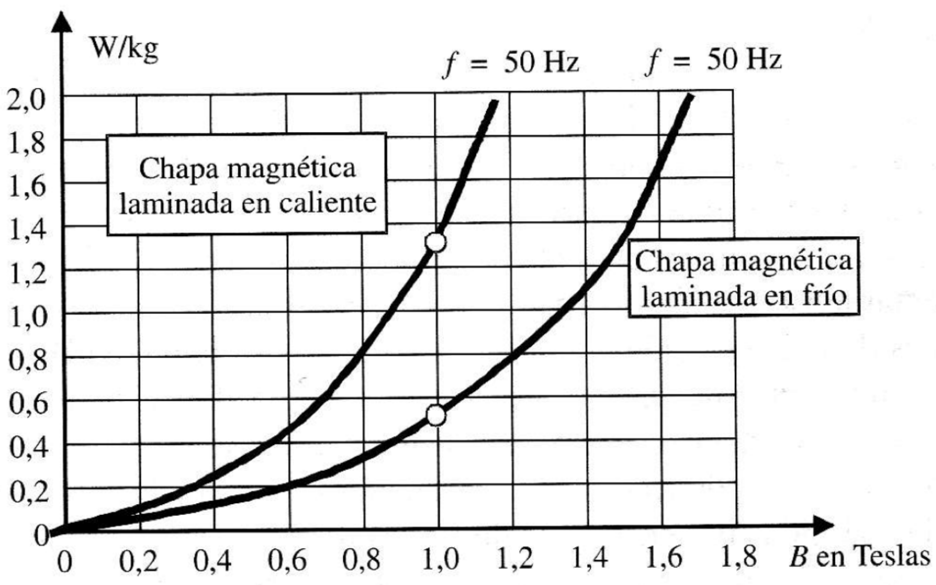


Figura 1.18. Curvas de pérdidas en el hierro.

MAGNETIC ENERGY

Eddy current looses:

EJEMPLO DE APLICACIÓN 1.6

Las pérdidas en el hierro de una muestra de material ferromagnético son de 1.000 W a 50 Hz. Cuando se aumenta la frecuencia hasta 100 Hz, manteniendo la inducción constante, las pérdidas totales correspondientes han sido de 2.500 W. Calcular las pérdidas por histéresis y por corrientes de Foucault para ambas frecuencias.

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AC & MAGNETISM

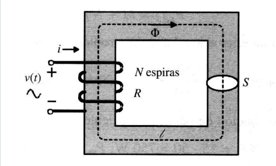
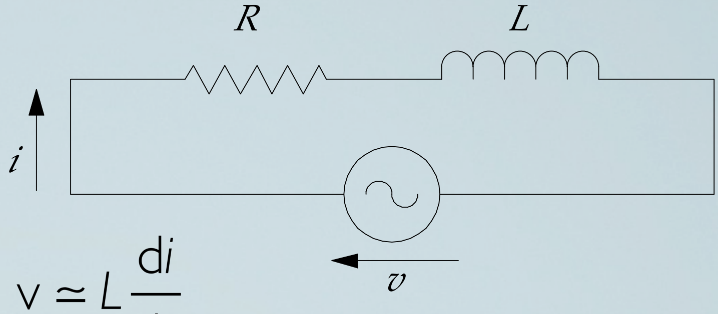


Figura 1.19. Bobina con núcleo de hierro.

$v_R = R\,i \qquad v_L = L\dfrac{di}{dt} \qquad v = v_R + v_L$

$$v = R\,i + L\frac{di}{dt}$$



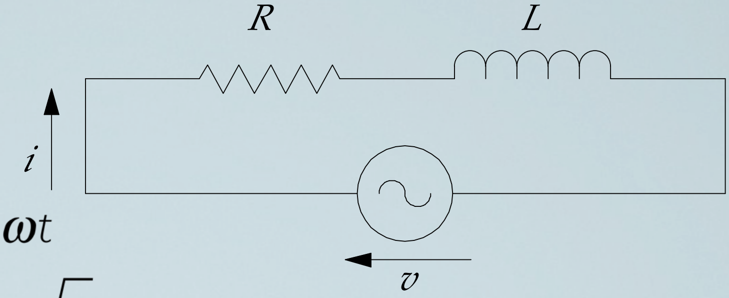
Como *Ri* es pequeño: $v \simeq L\dfrac{di}{dt}$

Self-inductance: $L = N\dfrac{d\Phi}{di}; \quad v = N\dfrac{d\Phi}{di}\dfrac{di}{dt} = N\dfrac{d\Phi}{dt}$

Magnetic flux: $d\Phi = \dfrac{1}{N}v\,dt; \quad \Phi = \dfrac{1}{N}\int v\,dt$

AC & MAGNETISM

Alternative voltage:



$$v = V_m \cos\omega t = \sqrt{2}\,V\cos\omega t$$

$$\Phi = \frac{1}{N}\int\sqrt{2}\,V\cos\omega t\,dt = \frac{\sqrt{2}\,V}{N\,\omega}\,\text{sen}\,\omega t = \Phi_m\,\text{sen}\,\omega t$$

$$\Phi_m = \frac{\sqrt{2}\,V}{N\,\omega}$$

$$\mathscr{F}_m = \Phi_m\mathscr{R}$$

$$\mathscr{F} = N\,i$$

$$N\,i_m = \Phi_m\mathscr{R}\,; \quad N\,i_m = \frac{\sqrt{2}\,V}{N\,\omega}\mathscr{R}$$

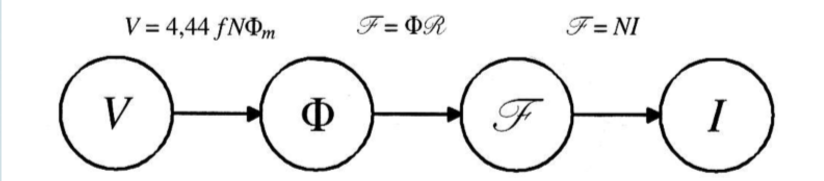


Figura 1.21. Sucesión de efectos en una bobina alimentada con **c.a.**

AC & MAGNETISM

Core with losses:

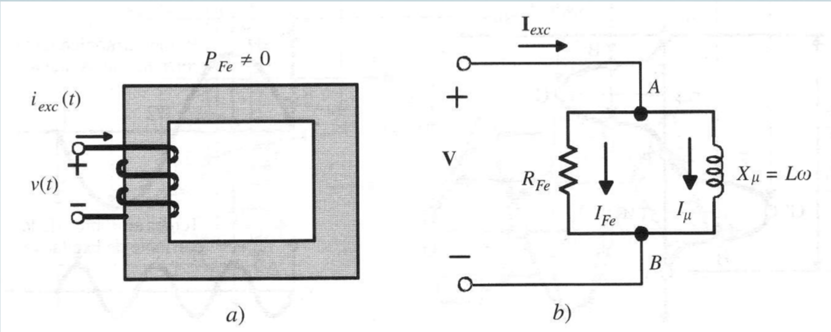
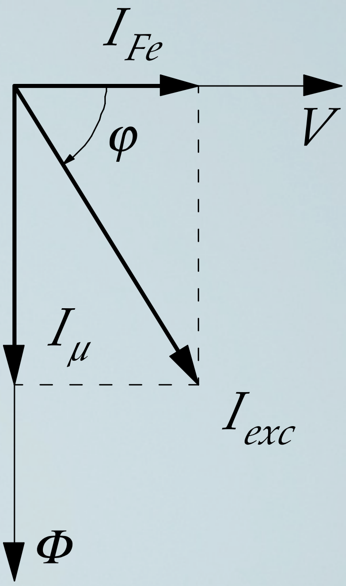


Figura 1.25. Circuito equivalente de una bobina con núcleo de hierro con pérdidas.



50 Hz : $\omega = 2\pi f$

$$X = L\omega; \quad \frac{1}{Z} = \frac{1}{R_{Fe}} + \frac{1}{jX_\mu} = \frac{1}{R_{Fe}} + \frac{1}{jL\omega}$$

$$P_{Fe} = V\, I_{Fe}$$